Science and teaching: Two-dimensional signalling in the academic job market

Preliminary version. Please do not cite without permission.

Andrea Schneider*
Helmut Schmidt University, 22043 Hamburg, Germany

Abstract
Post-docs signal their ability to do science and teaching to get a tenure giving universities the possibility to separate highly talented agents from the low talented. This paper shows that, if time constraints are binding, under weak conditions separating equilibria do not exist impliying that universities can partially separate some but not all types. Signalling efforts in these partial separating equilibria are higher than in a case where post-docs signal their science or teaching ability questioning the efficiency of the current two-dimensional academic appointment system.

Keywords: Multi-dimensional signalling; Academic job market; Teaching and Research

JEL classification: I23; D82; J41

1 Introduction

Not later than the 19th century the Germans know the concept of the unity of research and teaching. This idea of Wilhelm von Humboldt has mainly influenced especially the German higher education system and is still present today. On the other hand post-docs and professors often rail against the double burden of such a system. These conflicting argumentations in mind economists study the optimal design of the university system (e.g. Del Rey 2001, De Fraja and Valbonesi 2008 or Gautier and Wauthy 2007) as well as their optimal labour contract behaviour

*Corresponding author. Tel.: +49-40-6541-3725; Fax +49-40-6541-3409; E-mail: andrea.schneider@hsu-hh.de.

(Walckiers 2008). In line with the second part of literature the present paper analyses the possibility of separating highly productive agents from the low-productive ones in a model where post-docs can signal their ability to do science and teaching to get a tenure.
Argumenting in line with a job market sinalling model it is necessary to mention the work of Michael Spence. Spence as the father of signalling models shows education can be an efficient signal to correct asymmetric information in the job market. It's due to him that we know about the existence of signalling equilibria (Spence 1973; Spence 1974).[1] In contrast to Spence who mainly deals with the existence of equilibria Cho and Kreps (1987) rank equilibria. They implement an intuitve criterion to eliminate equilibria that are built on unplausible out-of-equilibrium beliefs. This stronger equilibrium concept is finally the basic equilibrium concept of the present paper.
Up to here all concepts work in a one-dimensional world. Thus, agents send a one-dimensional signal. Since future professors produce a two-dimensional output consisting of science and teaching a multi-dimensional set up is needed. Unfortunately, papers on multi-dimensional signalling are rare. One of the first is by Rochet and Quinzii (1985). This paper analyses in a formal way the difference between the one- and multi-dimensional signalling set up. Assuming a separable cost structure they give necessary conditions for the existence of a separating equilbrium. In the same kind of model Engers (1987) focuses on pareto-dominant separating equilibria. Armstrong and Rochet (1999) simplify conditions that are necessary to ensure a separating equilibrium by assuming a discrete type distribution. This is also an assumption of the present model. A current paper by Kim (2007) is of interest as well because it analyses time binding constraints in a two-dimensional job market signalling model.
The aim of this paper is to analyse separating equilibria in a two-dimensional signalling model that describes the academic job market. Post-docs that differ in their ability to do research and teaching can signal both talents to get a tenure. As one result separating equilibria of the two-dimensional case can vanish with time binding constraints. This always happens if teaching (science) productivity of the highly talented is higher than the science (teaching) productivity of the low-talented. Nevertheless, implying the concept of partial separating equilibria it can be shown that under weak conditions there is at least one partial separating equilibrium. More precisely, agents that are highly productive in both outputs send the same signal like the type that is high-talented with respect to the output that is more prefered by the universities. What is mportant for policiy implications is that the signalling effort in the partial separating equilibrium - although it is

[1]While Spence's first paper focuses on the general existence of signalling equilibria the second paper highlights the different market forms.

smaller than in the two-dimensional separating equilibrium - is higher than in the one-dimensional separating equilibrium. This is of interest if signalling only has an effect on costs but not on productivity as it is the case in Spence (1973) and also in the present model. Thus, if time constraints are binding in the academic job market it could be more efficient to let post-docs only signal on the output that is more prefered by the universities. Under time binding constraints universities can only distinguish highly productive from low-productive types in one dimension just like in the one-dimensional case. Having this in mind there is no argument for the two-dimensional signalling process that is currently observed in reality. It just implies additional signalling effort. Only weak conditions concerning the ranking of the productivity parameters are necessary to make a separating equilibrium under time binding constraints impossible in the two-dimensional case. Thus, agents that are highly-talented in both outputs can not be identified by universities.
The remaining paper is structured as follows: Section 2 sets the basic model. The existence of equilibria is analysed in section 3. While section 3.1 focuses on the one-dimensional case that goes in line with Spence (1973) section 3.2 extends the analysis to the two-dimensional case. In this part the paper distinguishes between a situation where time constraints are not binding resulting in the unique separating equilibrium is most efficient for the universities and a situation where time constraints are binding which may lead to the vanishing of the separating equilibrium. In the second case the existence of partial separating equilibria where some types of agents can be separated while others play the same stratgey is analysed. Section 4 concludes.

2 The model

Assume a competitive academic job market with a unit mass of academics. Each university graduate produces science and teaching which requires specific unobservable abilities.[2] Scientists as well as other labourers are not identical but vary in their abilities. There are four types $ij \in \{HH, HL, LH, LL\}$ of future professors.[3] While i denotes research productivity j describes the teaching productivity. Both productivities can be high (H) or low (L). Future professors can signal both abilities: Science s_{ij} and teaching t_{ij}, $i, j \in \{L, H\}$. As in Spence (1973) signals do not have any influence on productivity. Agents use the signals to influence the universities' beliefs on their abilities. Thus, the pre-tenure research and teaching outputs serve as a signal for post-tenure productivities. However, there is a time binding constraint $s_{ij} + t_{ij} = \bar{l}$. Signalling effort can not be higher than the

[2]In the remaining paper 'research', 'science' and 'publishing' are synonymously used.
[3]This notation follows Walckiers (2008).

available time and therefore is limited. I assume that the agents' ij cost function is:

$$c_{ij}(s_{ij}, t_{ij}) = \frac{s_{ij}}{\theta_i^s} + \frac{t_{ij}}{\theta_j^t}, \tag{1}$$

where θ_i^s and θ_j^t, $i, j\{L, H\}$, are the productivities of science and teaching respectively. Clearly, $\theta_H^k > \theta_L^k, k \in \{s, t\}$ holds. For simplicity I also assume $\theta_L^k \geq 1$, $k \in \{s, t\}$. Implicitly I assume that the research (teaching) productivity is independent of the ability to teach (research). The fraction of type ij agents in the population is denoted by α_{ij}. The distribution of the types is common knowledge. Universities compete on prospective professors. However, they face asymmetric information and can only form beliefs on the agents' abilities via signals.
The profit of a university is $\pi((\theta_i, \theta_j), w) = \theta_i^s + \theta_j^t - w$, where w is the wage paid to the agent. The competition of the academic job market implies that universities make a profit of zero and therefore wages are given by productivities. Thus, the equilibrium wage offered by the universities is w^*

$$w^* \equiv E[\alpha_{ij}(\theta_i^s + \theta_j^t)] \tag{2}$$

where E is the expectation operator.

Although pre-tenure publishing and teaching do not influence the productivity universities can condition wage offers on the pre-tenure science and teaching output. The optimal decision of a prospective professor of type ij is

$$\max_{s_{ij}, t_{ij}} U_{ij} = E[w_{ij} - (\frac{s_{ij}}{\theta_i^s} + \frac{t_{ij}}{\theta_j^t})]. \tag{3}$$

subject to $s_{ij} + t_{ij} \leq \bar{l}$.
In section 3 I will analyse equilibria of this signalling model.

3 Signalling in the academic job market

First I focus on signalling equilibria when universities are only interested in science (section 3.1). This analysis goes in line with the signalling model of Spence (1973). Afterwards in section 3.2 I analyse a two-dimensional signalling model where agents signal on science and teaching. In both cases the main question is if there are separating equilibria where signalling can help to solve inefficient results caused by asymmetric information. Therefore, pooling equilibria are only analysed in the margin.

Under incomplete information there is need for a definition of a perfect Bayesian equilibrium.

Definition 1 *Perfect Bayesian Equilibrium (PBE): A PBE is a set that consists of a signal* $(s^*_{ij},^* t_{ij})$ *for each type of agent* $ij \in \{HH, HL, LH, LL\}$ *and a wage offer* $w_{ij}(s^*_{ij}, t^*_{ij})$ *used by the universities.*
For each signal (s^*_{ij}, t^*_{ij}) *the universities make zero profits given the belief* $\mu(ij|(s_{ij}, t_{ij}))$ *about which types could have sent* (s_{ij}, t_{ij})*.*
Each type ij *maximises his utility by choosing* (s^*_{ij}, t^*_{ij}) *given the wage offer* w_{ij} *of the university.*
The university's belief must be consistent with Bayes' rule and with the agent's strategy: $\mu(ij|(s_{ij}, t_{ij})) = \frac{\alpha_{ij}}{\sum_{ij} \alpha_{ij}}$ *.*

Therefore, one can distinguish between a separating equlibrium and a pooling equilibrium. In the first case all types send different signals, i.e. $(s^*_{ij}, t^*_{ij}) \neq (s^*_{i'j'}, t^*_{i',j'})$ if $ij \neq i'j'$. In the second case the signal is identical for all types, i.e. (s^*_{ij}, t^*_{ij}), $\forall\ i, j \in \{H, L\}$. In contrast to a model set up with two different types of agents that is normally used, in the present model there is also the posibility for an equilibrium in which some but not all agents send the same signal. Such a perfect Bayesian equilibrium will be called a partial separating equilibrium.

3.1 One-dimensional signalling

Let us assume for the moment universities are only interested in science and not in teaching. In this case there is no value of teaching and therefore no agent sends a teaching signal. Thus, type HH and HL can be interpreted as one type denoted by H. The same applies to LH and LL. This low productivity type is denoted by L.[4] Then the fraction of the high productivity type is $\alpha_H \equiv \alpha_{HH} + \alpha_{HL}$ and the fraction of agents with low productivity is $\alpha_L \equiv \alpha_{LH} + \alpha_{LL}$.
Under complete information the high productivity type would earn a wage of θ^s_H while the type with low productivity gets $\theta^s_L < \theta^s_H$. Since pre-tenure publishing only implies a cost effect but no effect on productivity both types do not publish anything under complete information. Under incomplete information one can distinguish between a pooling and a separating equilibrium; the partial separating equilibrium is irrelevant in the case of two different types of agents.

[4]Clearly, in this two type case the cost and wage structure satifies the well known Spence-Mirrlees *single crossing property* condition, i.e. the two-types' $w - s_i-$indifference curves with $i \in \{H, L\}$ have only one point of intersection.

Proposition 1 *Given a two type signalling game where future professors can have high or low productivity of publishing (θ_H^s or θ_L^s) and the universities' wage offer $w(s)$ depending on the research signal s there is a unique separating equilibrium*

$$s_H^* = \theta_L^s(\theta_H^s - \theta_L^s), \; s_L^* = 0$$

$$w(s_H^*) = \theta_H^s, \; w(s_L^*) = \theta_L^s$$

$$\mu(H|s \geq s_H^*) = 1, \; \mu(L|s < s_H^*) = 1.$$

This is a result of the standard signalling theory: In a sepearting equilibrium there is no incentive for the type with low productivity to invest in publishing because this has just a cost effect but no impact on productivity. Therefore, an agent with high productivity must publish exactly the amount that ensures type L does not mimic him, i.e. the incentive compatibility condition $w(s_L) - c_L(s_L) \geq w(s_H) - c_L(s_H)$ must hold. This directly gives s^*. Beside this, it is possible that the time constraint is binding, i.e. $s_H^* > \bar{l}$. Then the agent with the high productivity can not publish enough to prevent mimicing of the low-productive type.[5]

Of course, all results persist if universities are solely interested in teaching. In this case just replace s by t in the previous analysis and redefine $\alpha_H \equiv \alpha_{HH} + \alpha_{LH}$ and $\alpha_L \equiv \alpha_{HL} + \alpha_{LL}$ respectively.

3.2 Two-dimensional signalling

Results on the interdependency of science and teaching seem to be amibiguous: A higher load of teaching (and also administrative work) reduces publication output since time to do research can not be used to teach (Mitchell and Rebne 1995). Although teaching can enhance research (Becker and Kennedy 2005) there is no general evidence that good researchers are also good teachers. In contrast economists prefer doing reseach to teaching (Allgood and Walstad 2005).

Because of the complex linking of science and teaching, I do not make any additional assumptions on the distribution of the four types of agents.[6] Nevertheless note, if both talents are substitutes (complements) α_{HL} and α_{LH} are high (smal) while α_{HH} and α_{LL} are small (high).[7]

[5] In addition there is also a unique pooling equilibrium where nobody signals, i.e. $s_H^* = 0$

[6] Although there is no clear evidence that research and teaching are complements on the individual perspective level both act complementary on the university level. For a meta-analysis on this topic see Hattie and Marsh (1996).

[7] Gottlieb and Keith (1997) find that the connection between research and teaching is not just substitutive or complementary but more complex. In detail they show that research can positively affect research but attributes of teaching negatively impact research.

This subsection firstly analyses the separating equilbrium in the two- dimensional case, then it shows that under some conditions (more precisely, if Assumption 1 holds) time constraints make a separating equilibrium imposssible. Nevertheless, if agents send a two-dimensional signal there is always at least one partial separating equilibrium. In this equilibrium type HH sends the same signal like the type that is highly talented with respect to the output that is more prefered by the universities.

Time constraint not binding

Proposition **2** *If agents signal science and teaching ability via (s_{ij}, t_{ij}), universities offer wages $w(s_{ij}, t_{ij})$ and the time constraint is not binding, i.e. $s^*_{ij} + t^*_{ij} \leq \bar{l}$, there is a separating equilibrium*

$$s^*_{ij} = \begin{cases} \theta^s_L(\theta^s_H - \theta^s_L), & i = H \\ 0, & i = L \end{cases} \quad \text{and} \quad t^*_{ij} = \begin{cases} \theta^t_L(\theta^t_H - \theta^t_L), & j = H \\ 0, & j = L \end{cases}$$

$$w(s^*_{ij}, t^*_{ij}) = \theta^s_i + \theta^t_j, \ \forall i, j \in \{H, L\}$$

$$\mu(i, j = H | k_{ij} \geq \theta^k_L(\theta^k_H - \theta^k_L)) = 1 \text{ and } \mu(i, j = L | k_{ij} < \theta^k_L(\theta^k_H - \theta^k_L)) = 1$$

where $k \in \{s, t\}$.

The detailed proof of Proposition 2 is given in the appendix page 14. The basic idea is to derive conditions under which type ij has no incentive to mimic type $i'j'$ for all $i, i', j, j' \in \{H, L\}$. Although these conditions are fulfiled by a continuum of signal combinations (s_{ij}, t_{ij}) there is only a unique signal for each type that maximises utility. Caused by additive linearity of costs and productivities the signals in the two-dimensional PBE equal in each of the two components the signals arising in the one-dimensional case.[8]

Time constraint binding

What happens if time constraints are binding, i.e. if type HH can not play his strategy of the separating equilibrium of Proposition 2? Formally, what is PBE if

[8] The same argumentation as in the one-dimensional case leads to a pooling PBE where nobody signals, i.e. all agents' strategy is $(s^* = 0, t^* = 0)$. There is also the possibility for partial separating PBEs in the present case. However, universities are interested in the "real" type of the agent. So, the most efficient situation is the separating one. I pay more attention to the partial separating PBEs in the next subsection where time constraints play a crucial role.

$s^*_{HH} + t^*_{HH} > \bar{l}$ holds? For simplicity I assume that $\theta^k_L(\theta^k_H - \theta^k_L) \leq \bar{l}$, $k \in \{s,t\}$, holds. This gurantees that the separating equilibrium of the one-dimensional case exists. If this is not fulfiled only the pooling equilibrium where nobody signals remains.
As a key mechanism of a separating equilibrium the highly talented agent separates himself by signalling so much that there is no incentive of the low-talented agent to mimic him. This is possible because of the difference in costs. However, if there are not only one but two signals the signalling effort increases[9] and may become too high to be realised in the time given.
Before discussing the main result of this section I make an assumption about the ranking of the productivity parameters that is crucial for the remaining analysis.

Assumption **1** *The ranking of the productivity parameters fulfils*

$$\theta^s_H \geq \theta^t_L$$

and

$$\theta^t_H \geq \theta^s_L.$$

By defintion $\theta^k_H > \theta^k_L$, $k \in \{s,t\}$ always holds. So, for both activities the highly talented agent is more productive than the agent with low productivity. However, nothing is known about the ranking of the productivity parameters comparing both activities. Assumption 1 requires that agents that are highly productive doing one activity are more productive than agents doing the other activity with low talent. Or, the other way round, Assumption 1 is violated if the universities' benefit from one output is so high that producing this output by a low-productive agent is better than producing the other output by a high-productive agent.

Proposition **3** *If agents signal their abilities to do science (s_{ij}) and teaching (t_{ij}), universities offer wages $w(s_{ij}, t_{ij})$, the time constraint is binding, i.e. if in Proposition 2 $s^*_{HH} + t^*_{HH} > \bar{l}$, and Assumption 1 holds, there is no separating equilibrium.*
If Assumption 1 does not hold the separating equilibrium from Proposition 2 is destroyed but there is another separating equilibrium.

For an illustration of a situation where Assumption 1 holds see Figure 1.[10] The figure describes the incentive compatibilty constraints of type HH. All strategies

[9] In the present model the signalling effort in the two-dimensional case is exactly the sum of the two one-dimensional signalling models where the agent signals on teaching or science. However, this result is driven by the additive structure of productivity and costs.
[10] The figure refers to parameter setting $\theta^s_L = 2$, $\theta^s_H = \theta^t_L = 3$ and $\theta^t_H = 4$.

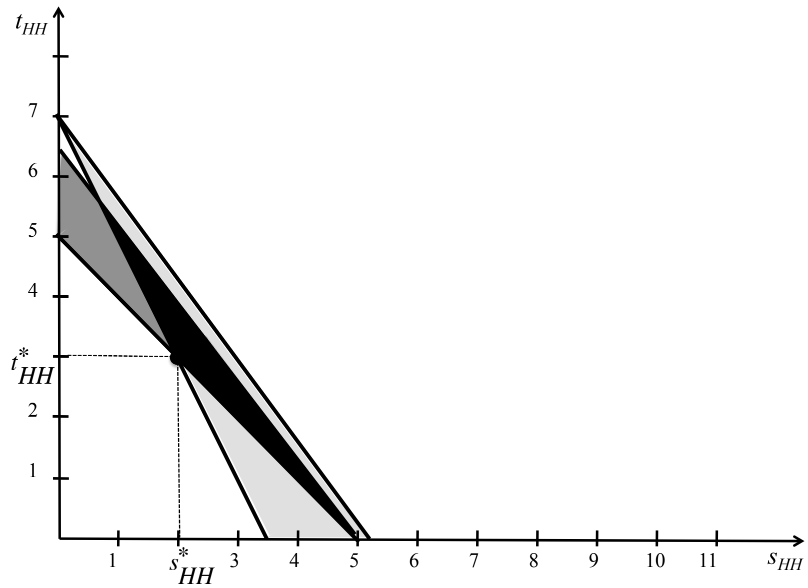

Figure 1: Incentive compatibility constraints for type HH when Assumption 1 holds.

in the light-grey triangle prevent HH from mimicing LH and vice versa. The dark-grey triangle consists of all s-t-combinations that prevent HH from mimicing HL and vice versa. The black triangle therefore gives all strategies that fulfil both conditions. The strategy (s^*_{HH}, t^*_{HH}) is the equilibrium strategy. The key idea here is as follows: Because of the pure cost effect of signalling HH realises a cost minimal combination that is tangent to the black triangle at its lower bound. The lower bound of the light-grey triangle has a slope of $-(\theta^t_H/\theta^s_L)$. The lower bound of the dark-grey triangle has a slope of $-(\theta^t_L/\theta^s_H)$. Since the slope of HH's cost function is $-(\theta^t_H/\theta^s_H)$ and therefore meets the condition $-(\theta^t_H/\theta^s_L) < -(\theta^t_H/\theta^s_H) < -(\theta^t_L/\theta^s_H)$ strategy (s^*_{HH}, t^*_{HH}) becomes the cost minimal strategy that fulfils both incentive compatibility constraints. However, if the euqilibrium strategy of type HH, i.e. (s^*_{HH}, t^*_{HH}), is unrealisable because of time constraints there is no other strategy that fulfils the incentive compatibility constraints of type HH. So, if Assumption 1 holds there is no spearating PBE.

In contrast, Figure 2 shows a situation in which Assumption 1 does not hold.[11] The grey area describes all strategies of HH that fulfil both incentive compatibility constraints. Contrary to Figure 1 a decrease in the available time from $\bar{l}_1$ to $\bar{l}_2$ shifts the separating PBE from HH_1 to HH_2. Thus there is still the possibility of separating the different types of agents.

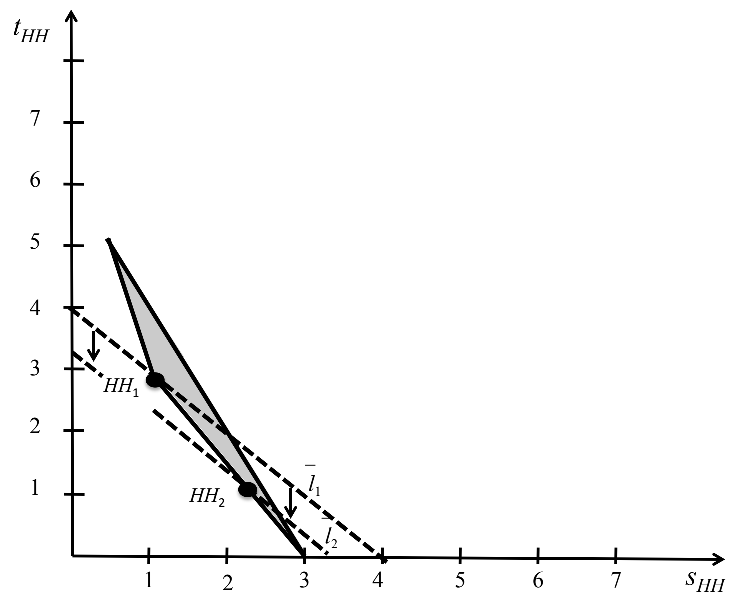

Figure 2: Incentive compatibility constraints for type HH when Assumption 1 is not fulfiled.

If Assumption 1 does not hold univeristies can only separate some but not all types of agents. However, in the two-dimensional case there is always - depending on the productivity parameters - at least one partial separating equilibrium.

PROPOSITION **4** *If agents signal their abilities to do science* (s_{ij}) *and teach* (t_{ij}), *universities offer a wage* $w(s_{ij}, t_{ij})$ *equal to the expected productivity there are two partial separating equilibria.*

[11]The figure referes to the parameter setting $\theta^s_L = 1$, $\theta^s_H = 2$, $\theta^t_L = 3$ and $\theta^t_H = 4$.

If $\theta^s_L\theta^t_H \geq \theta^s_H\theta^t_L$ holds there is a partial PBE where strategies of the prospective professors are:

$$(s^*_{LL}, t^*_{LL}) = (0,0), \quad (s^*_{HL}, t^*_{HL}) = (\theta^s_L(\theta^s_H - \theta^s_L), 0) \quad \text{and}$$
$$(s^*_{(LH,HH)}, t^*_{(LH,HH)}) = (0, \theta^t_L C1_{(LH,HH)})$$

with $C1_{LH,HH} \equiv \frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_H - (1 - \frac{\alpha_{LH}}{\alpha_{LH}+\alpha_{HH}})\theta^s_L + \theta^t_H - \theta^t_L$.

If $\theta^s_H\theta^t_L \geq \theta^s_L\theta^t_H$ holds there is a partial separating PBE where strategies of the prospective professors are:

$$(s^*_{LL}, t^*_{LL}) = (0,0), \quad (s^*_{LH}, t^*_{LH}) = (0, \theta^t_L(\theta^t_H - \theta^t_L)) \quad \text{and}$$
$$(s^*_{(HL,HH)}, t^*_{(HL,HH)}) = (\theta^s_L C1_{(HL,HH)}, 0)$$

with $C1_{(HL,HH)} = \theta^s_H - \theta^s_L + \frac{\alpha_{HH}}{\alpha_{HL}+\alpha_{HH}}\theta^t_H - (1 - \frac{\alpha_{HL}}{\alpha_{HL}+\alpha_{HH}})\theta^t_L$.

In cause of clear arrangement Proposition 4 only denotes strategies of the prospective professors.[12] The wage setting of the universities is for the separated types equal to the wage setting of Proposition 2. The pooled types are paid by average productivities. Thus in the first partial separating PBE it is $w_{(LH,HH)} = \frac{\alpha_{LH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_L + \frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_H + \theta^t_H$ and in the second partial separating equilibrium it is $w_{(HL,HH)} = \theta^s_H + \frac{\alpha_{HL}}{\alpha_{HL}+\alpha_{HH}}\theta^t_L + \frac{\alpha_{HH}}{\alpha_{HL}+\alpha_{HH}}\theta^t_H$.
The detailed proof can be found in the appendix on page 19. In the first partial separating PBE universities can distinguish between LL, HL and (LH, HH), i.e. they can not separate type LH from HH. In the second partial separating PBE universities can separate LL from LH and (HL, HH) but not types HL and HH. The key arrangement of the proof of the first partial separating PBE (and analogously of the second one) is as follows: Type LL does not signal because of the pure cost effect. Type HL playes his strategy from the one-dimensional case to prevent LL from mimicing. Then the incentive compatibility constraints of (LH, HH) not to mimic LL or HL and vice versa are calculated. This results in the equilibrium strategy for (LH, HH).

Figure 3 illustrates the necessary condition of the existence of the first partial separating PBE $(LL, HL, (LH, HH))$, i.e. $\theta^s_L\theta^t_H \geq \theta^s_H\theta^t_L$.[13] In part (a) of Figure 3

[12]Proposition 4 only describes two partial separating equilibria. There is also the possibility of other partial separating equilibria, e.g. of $((LL, LH), HL, HH)$. Nevertheless, universities try to identify the highly productive agents. Thus the partial separating PBEs of Proposition 4 are the one of interest.

[13]Clearly, an analogous argumentation holds for condition $\theta^s_H\theta^t_L \geq \theta^s_L\theta^t_H$ and the second partial separating PBE.

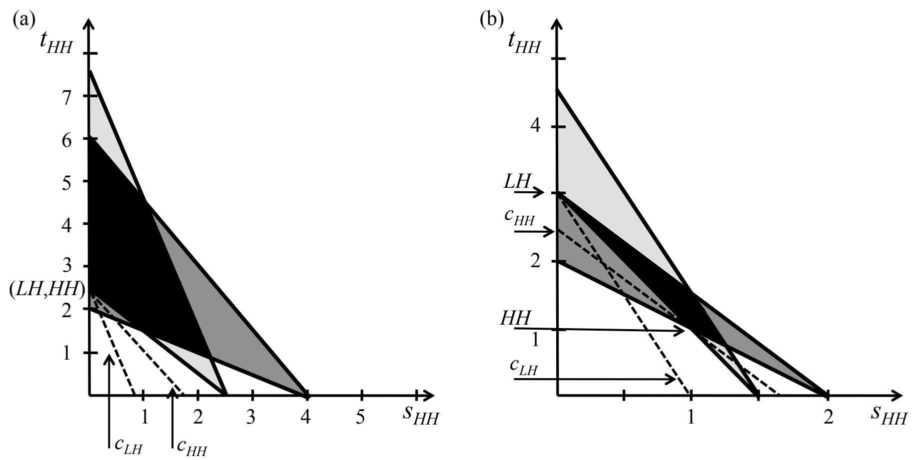

Figure 3: Incentive compatibility constraints for (LH, HH) in the first partial separating PBE (a) when $\theta_L^s\theta_H^t \geq \theta_H^s\theta_L^t$ holds and (b) if this condition is not fulfiled

it is $\theta_L^s\theta_H^t \geq \theta_H^s\theta_L^t$ and both minimal cost functions of the pooled types, i.e. c_{LH} and c_{HH}, are tangent to the black array that consists of all strategies which meet the incentive compatibility constraints at point (LH, HH).[14] This s-t-combination is the strategy LH and HH play in the first partial separating equilibrium. In part (b) it is $\theta_L^s\theta_H^t < \theta_H^s\theta_L^t$.[15] Thus, the cost function of HH, i.e. c_{HH}, runs 'too flat'. The minimal cost function of type HH is tangent to the black area where the incentive compatibility constraints are fulfiled at point HH. Since the minimal cost function of type LH is tangent to the black array at point LH there is no pooling equilibrium strategy for both types and so no partial separating PBE.

[14]Part (a) of the figure refers to parameter values $\theta_L^s = \theta_L^t = 1$, $\theta_H^s = 2$ and $\theta_H^t = 3$. More precisely, optimal strategies should be labeled $(s^*_{(LH,HH)}, t^*_{(LH,HH)})$. However, caused by clarification I label the strategy with the type.

[15]Part (b) of Figure 3 refers to $\theta_L^s = 1$, $\theta_H^s = \theta_L^t = 2$ and $\theta_H^t = 3$.

As a first result one can see that both partial separating PBE can only co-exist if $\theta^s_H\theta^t_L = \theta^s_L\theta^t_H$ holds. One example for such a situation is the symmetric case, where low (high) productivity of science equals low (high) productivity of teaching, i.e. $\theta^s_H = \theta^t_H$ and $\theta^s_L = \theta^t_L$. Thus, universities do not have a clear preference for the one or the other output. Assuming that the highly productive agents are the critical one and therefore normalising the productivities of the low-talented to one, i.e. $\theta^s_L = \theta^t_L = 1$, the first partial separating PBE only exists if teaching productivity of the highly talented is higher than his research productivity. Analogously, if the contrary appraisement holds the second partial separating PBE appears. In general an angent that is good in teaching and science pooles with the type that is highly-talented in the output that is more prefered by the universities. This strengthens the argument of Becker (1975, 1979) that the professors' research and teaching output positively react on an increase in pecuniary returns.
Secondly, it is clear that without time constraints always at least one of the partial PBEs exists. However, in this case they are less interesting because the separating PBE is more efficient. However, if time constraints are too strong even a partial separating equilibrium maybe does not exist and only the pooling PBE where nobody signals remains.
Thirdly, especially if high-talented agents are of interest signalling effort in the partial separating equilibrium is higher than in the one-dimensional case. The effort of type HH in the first partial separating equilibrium is $\theta^t_L C1_{(LH,HH)} = \theta^t_L \frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}}(\theta^s_H - \theta^s_L) + \theta^t_L(\theta^t_H - \theta^t_L)$. This is clearly higher than his effort in the one-dimensional case, i.e. higher than $\theta^t_L(\theta^t_H - \theta^t_L)$. Thus, the partial separating equilibrium requires higher signalling effort although there is no additional information for the universities with respect to the type that is highly talented in science and teaching. An analogous argumentation applies for the second partial separating equilibrium.

4 Conclusion

The output of post-docs and professors consists, beside the administrative one that is not mentioned here, of science and teaching. In general universities are interested in both outputs and assign a tenure contract only to those post-docs that are highly talented in both activities. However, since talent is a private information a job market signalling model ła Spence arises. Post-docs signal their ability of science and teaching to get a tenure.
As Spence (1973) has shown in the one-dimensional case signalling can also in the two-dimensional case separate highly talented and low talented agents. So it solves the inefficiency problem of asymmetric information. Unfortunately, the

highly productive agents need a signalling effort to separate themselves from the low-productive types and this effort increases in the two-dimensional case. Considering this, time constraints attract notice.
If time constraints are binding and the science (teaching) productivity of the high-talented is higher than the teaching (science) productivity of the type with low talent a separating equilibrium can not exist in the two-dimensional case. The required assumption is quite weak as it just says that universities should not prefer one output over the other regardless wether the first is created by a high- or low-productive person.
In addition I show that even if the separating equilibrium is destroyed by time constraints there is always at least one partial separating equilibrium where some types can be separated while others pool on the same strategy. More precisely, if the university prefers science to teaching a partial separating equilibrium exists where universities can separate types with high or low research productivity. However, they do not know if an agent with high research productivity is also highly talented in teaching. This is the same result as in the one-dimensional case. Regrettably, the signalling effort that only implies a pure cost effect is higher in the two-dimensional partial separating equilibrium than in the one-dimensional separating one. Corresponding to real life, the two-dimensional signalling system that is currently used in academic admission processes is inefficient if time constraints are binding. In such a situation universities can not identify both talents of he post-doc but only one. The identifiable talent is the one they value more. Then universities can ease requirements on post-docs and can let them - without loosing information - just signal on science or teaching.

Appendix

Proof of Proposition 2:
In a separating PBE universities pay an agent ij a wage equal to his productivity. Thus, $w(s^*_{ij}, t^*_{ij}) = \theta^s_i + \theta^t_j$ holds.
This directly gives $(s^*_{LL}, t^*_{LL}) = (0, 0)$ as equilibrium signal of type LL. In a next step, signals of types HL and LH must meet the incentive compatibility constraints so that both types have no incentive to mimic LL and vice versa. This automatically prevents HH from mimicing LL.

Type HL does not mimic LL if

$$\begin{aligned}
w(s^*_{LL}, t^*_{LL}) - c_{HL}(s^*_{LL}, t^*_{LL}) &\leq w(s_{HL}, t_{HL}) - c_{HL}(s_{HL}, t_{HL}) \\
\Leftrightarrow \theta^s_L + \theta^t_L &\leq \theta^s_H + \theta^t_L - \frac{s_{HL}}{\theta^s_H} - \frac{t_{HL}}{\theta^t_L} \\
\Leftrightarrow \frac{1}{\theta^s_H} s_{HL} + \frac{1}{\theta^t_L} t_{HL} &\leq \theta^s_H - \theta^s_L
\end{aligned}$$

holds.
Analogously, LL does not mimic HL whenever

$$\begin{aligned}
w(s^*_{LL}, t^*_{LL}) - c_{LL}(s^*_{LL}, t^*_{LL}) &\geq w(s_{HL}, t_{HL}) - c_{LL}(s_{HL}, t_{HL}) \\
\Leftrightarrow \theta^s_L + \theta^t_L &\geq \theta^s_H + \theta^t_L - \frac{s_{HL}}{\theta^s_L} - \frac{t_{HL}}{\theta^t_L} \\
\Leftrightarrow \frac{1}{\theta^s_L} s_{HL} + \frac{1}{\theta^t_L} t_{HL} &\geq \theta^s_H - \theta^s_L
\end{aligned}$$

holds. Therefore the incentive compatibility constraint that prevents HL from mimicing LL and vice versa is

$$\frac{1}{\theta^s_H} s_{HL} + \frac{1}{\theta^t_L} t_{HL} \leq \theta^s_H - \theta^s_L \leq \frac{1}{\theta^s_L} s_{HL} + \frac{1}{\theta^t_L} t_{HL}.$$

A signal that maximises utility of type HL must lie on the lower bound which one can rewrite as

$$s_{HL} = \theta^s_L(\theta^s_H - \theta^s_L) - \frac{\theta^s_L}{\theta^t_L} t_{HL}.$$

Type HL will now choose the signal that fulfils this condition and minimises costs. Since costs are (taking the last equation into account)

$$\begin{aligned}
c_{s_{HL}, t_{HL}} &= \frac{s_{HL}}{\theta^s_H} + \frac{t_{HL}}{\theta^t_L} \\
&= \frac{\theta^s_L(\theta^s_H - \theta^s_L)}{\theta^s_H} - \frac{\theta^s_L}{\theta^s_H \theta^t_L} t_{HL} + \frac{1}{\theta^t_L} t_{HL} \\
&= \frac{\theta^s_L(\theta^s_H - \theta^s_L)}{\theta^s_H} + \underbrace{(1 - \frac{\theta^s_L}{\theta^s_H})\frac{1}{\theta^t_L}}_{>0} t_{HL}
\end{aligned}$$

the minimal cost combination is $t^*_{HL} = 0$ and therefore $s^*_{HL} = \theta^s_L(\theta^s_H - \theta^s_L)$. Type HL's strategy in the separating PBE is (s^*_{HL}, t^*_{HL}).

In the same way type LH does not mimic type LL if

$$\begin{aligned} w(s^*_{LL}, t^*_{LL}) - c_{LH}(s^*_{LL}, t^*_{LL}) &\leq w(s_{LH}, t_{LH}) - c_{LH}(s_{LH}, t_{LH}) \\ \Leftrightarrow \theta^s_L + \theta^t_L &\leq \theta^s_L + \theta^t_H - \frac{s_{LH}}{\theta^s_L} - \frac{t_{LH}}{\theta^t_H} \\ \Leftrightarrow \frac{1}{\theta^s_L} s_{LH} + \frac{1}{\theta^t_H} t_{LH} &\leq \theta^t_H - \theta^t_L \end{aligned}$$

holds.
Type LL does not mimic type LH if

$$\begin{aligned} w(s^*_{LL}, t^*_{LL}) - c_{LL}(s^*_{LL}, t^*_{LL}) &\geq w(s_{LH}, t_{LH}) - c_{LL}(s_{LH}, t_{LH}) \\ \Leftrightarrow \theta^s_L + \theta^t_L &\geq \theta^s_L + \theta^t_H - \frac{s_{LH}}{\theta^s_L} - \frac{t_{LH}}{\theta^t_L} \\ \Leftrightarrow \frac{1}{\theta^s_L} s_{LH} + \frac{1}{\theta^t_L} t_{LH} &\geq \theta^t_H - \theta^t_L \end{aligned}$$

is fulfiled. Taking both conditions together type LH has no incentive to mimic type LL and vice versa if

$$\frac{1}{\theta^s_L} s_{LH} + \frac{1}{\theta^t_H} t_{LH} \leq \theta^t_H - \theta^t_L \leq \frac{1}{\theta^s_L} s_{LH} + \frac{1}{\theta^t_L} t_{LH}$$

holds. Again LH chooses a signal on the lower bound given by the second part of the condition. Thus it is

$$t_{LH} = \theta^t_L(\theta^t_H - \theta^t_L) - \frac{\theta^t_L}{\theta^s_L} s_{LH}.$$

This in mind costs of type HL are given by

$$\begin{aligned} c_{LH}(s_{LH}, t_{LH}) &= \frac{s_{LH}}{\theta^s_L} + \frac{t_{LH}}{\theta^t_H} \\ &= \frac{\theta^t_L(\theta^t_H - \theta^t_L)}{\theta^t_H} + \frac{1}{\theta^s_L} s_{LH} - \frac{\theta^t_L}{\theta^s_L \theta^t_H} s_{LH} \\ &= \underbrace{(1 - \frac{\theta^t_L}{\theta^t_H}) \frac{1}{\theta^s_L}}_{>0} s_{LH} + \frac{\theta^t_L(\theta^t_H - \theta^t_L)}{\theta^t_H}. \end{aligned}$$

To minimise costs and therefore maximise utility given the wage $\theta^s_L + \theta^t_H$ type LH plays $s^*_{LH} = 0$ and $t^*_{LH} = \theta^t_L(\theta^t_H - \theta^t_L)$ in equlibrium.

With (s^*_{HL}, t^*_{HL}) and (s^*_{LH}, t^*_{LH}) type HL has no incentive to mimic type LH and vice versa cause

$$\begin{aligned}
w(s^*_{LH}, t^*_{LH}) - c_{HL}(s^*_{LH}, t^*_{LH}) &\leq w(s^*_{HL}, t^*_{HL}) - c_{HL}(s^*_{HL}, t^*_{HL}) \\
\Leftrightarrow \theta^s_L + \theta^t_H - \frac{\theta^t_L(\theta^t_H - \theta^t_L)}{\theta^t_L} &\leq \theta^s_H + \theta^t_L - \frac{\theta^s_L(\theta^s_H - \theta^s_L)}{\theta^s_H} \\
\Leftrightarrow 0 &\leq (\theta^s_H - \theta^s_L)^2
\end{aligned}$$

and

$$\begin{aligned}
w(s^*_{HL}, t^*_{HL}) - c_{LH}(s^*_{HL}, t^*_{HL}) &\leq w(s^*_{LH}, t^*_{LH}) - c_{LH}(s^*_{LH}, t^*_{LH}) \\
\Leftrightarrow \theta^s_H + \theta^t_L - \frac{\theta^s_L(\theta^s_H - \theta^s_L)}{\theta^s_L} &\leq \theta^s_L + \theta^t_H - \frac{\theta^t_L(\theta^t_H - \theta^t_L)}{\theta^t_H} \\
\Leftrightarrow 0 &\leq (\theta^t_H - \theta^t_L)^2
\end{aligned}$$

are always fulfiled.

In a last step one has to make sure that HH does neither mimic HL nor LH and vice versa. Type HH does not mimic HL whenever

$$\begin{aligned}
w(s^*_{HL}, t^*_{HL}) - c_{HH}(s^*_{HL}, t^*_{HL}) &\leq w(s_{HH}, t_{HH}) - c_{HH}(s_{HH}, t_{HH}) \\
\Leftrightarrow \theta^s_H + \theta^s_L - \frac{\theta^s_L(\theta^s_H - \theta^s_L)}{\theta^s_H} &\leq \theta^s_H + \theta^t_H - \frac{s_{HH}}{\theta^s_H} - \frac{t_{HH}}{\theta^t_H} \\
\Leftrightarrow s_{HH} + \frac{\theta^s_H}{\theta^t_H} t_{HH} &\leq \theta^s_H(\theta^t_H - \theta^t_L) + \theta^s_L(\theta^s_H - \theta^s_L)
\end{aligned}$$

holds.
Analogously, type HL has no incentive to mimic HH if

$$\begin{aligned}
w(s_{HH}, t_{HH}) - c_{HL}(s_{HH}, t_{HH}) &\leq w(s^*_{HL}, t^*_{HL}) - c_{HL}(s^*_{HL}, t^*_{HL}) \\
\Leftrightarrow \theta^s_H + \theta^t_H - \frac{s_{HH}}{\theta^s_H} - \frac{t_{HH}}{\theta^t_L} &\leq \theta^s_H + \theta^t_L - \frac{\theta^s_L(\theta^s_H - \theta^s_L)}{\theta^s_H} \\
\Leftrightarrow s_{HH} + \frac{\theta^s_H}{\theta^t_L} t_{HH} &\geq \theta^s_H(\theta^t_H - \theta^t_L) + \theta^s_L(\theta^s_H - \theta^s_L)
\end{aligned}$$

is fulfiled. Both conditions together are the incentive compatibility condition that prevents HH from mimicing HL and vice versa. Because of the pure cost effect of signalling the lower bound of the second condition, i.e.

$$\begin{aligned}
s_{HH} + \frac{\theta^s_H}{\theta^t_L} t_{HH} &= \theta^s_H(\theta^t_H - \theta^t_L) + \theta^s_L(\theta^s_H - \theta^s_L) \\
\Leftrightarrow s_{HH} &= \theta^s_H(\theta^t_H - \theta^t_L) + \theta^s_L(\theta^s_H - \theta^s_L) - \frac{\theta^s_H}{\theta^t_L} t_{HH} \qquad (4)
\end{aligned}$$

is a necassary condition for a separating PBE. However additionally, type HH does not have an incentive to mimic type LH and vice versa. Therefore,

$$\begin{aligned}
w(s^*_{LH}, t^*_{LH}) - c_{HH}(s^*_{LH}, t^*_{LH}) &\leq w(s_{HH}, t_{HH}) - c_{HH}(s_{HH}, t_{HH}) \\
\Leftrightarrow \theta^s_L + \theta^t_H - \frac{\theta^t_L(\theta^t_H - \theta^t_L)}{\theta^t_H} &\leq \theta^s_H + \theta^t_H - \frac{s_{HH}}{\theta^s_H} - \frac{t_{HH}}{\theta^t_H} \\
\Leftrightarrow \frac{\theta^t_H}{\theta^s_H} s_{HH} + t_{HH} &\leq \theta^t_H(\theta^s_H - \theta^s_L) + \theta^t_L(\theta^t_H - \theta^t_L)
\end{aligned}$$

and

$$\begin{aligned}
w(s_{HH}, t_{HH}) - c_{LH}(s_{HH}, t_{HH}) &\leq w(s^*_{LH}, t^*_{LH}) - c_{LH}(s^*_{LH}, t^*_{LH}) \\
\Leftrightarrow \theta^s_H + \theta^t_H - \frac{s_{HH}}{\theta^s_L} - \frac{t_{HH}}{\theta^t_H} &\leq \theta^s_L + \theta^t_H - \frac{\theta^t_L(\theta^t_H - \theta^t_L)}{\theta^t_H} \\
\Leftrightarrow \frac{\theta^t_H}{\theta^s_L} s_{HH} + t_{HH} &\geq \theta^t_H(\theta^s_H - \theta^s_L) + \theta^t_L(\theta^t_H - \theta^t_L)
\end{aligned}$$

must hold. Both condtions together are the incentive compatibility constraint that prevent HH from mimicing LH and vice versa. Cause of the pure cost effect of signalling the lower bound of the second condition, i.e.

$$\begin{aligned}
\frac{\theta^t_H}{\theta^s_L} s_{HH} + t_{HH} &= \theta^t_H(\theta^s_H - \theta^s_L) + \theta^t_L(\theta^t_H - \theta^t_L) \\
\Leftrightarrow s_{HH} &= \theta^s_L(\theta^s_H - \theta^s_L) + \frac{(\theta^t_H - \theta^t_L)\theta^s_L\theta^t_L}{\theta^t_H} - \frac{\theta^s_L}{\theta^t_H} t_{HH} \qquad (5)
\end{aligned}$$

is a necessary condition for a PBE. In a separating PBE type HH neither mimics HL nor LH. Thus, conditions (4) and (5) must hold. Both linear functions describe the lower bound of the area that fulfils both incentive compatibility constraints. Because of the pure cost effect of signalling the optimal strategy is element of this lower bound. To make sure that the optimal startegy is unique the slope of this lower bound must be unequal to the slope of the cost function of HH.[16] The cost function of type HH is $c_{HH}(s_{HH}, t_{HH}) = (s_{HH}/\theta^s_H) + (t_{HH}/\theta^t_H)$. So, the slope of this function in a s-t-area is $-(\theta^t_H/\theta^s_H)$. As the slope of equation (4) in such an area is $-(\theta^t_L/\theta^s_H)$ and the slope of equation (5) is $-(\theta^t_H/\theta^s_L)$ there is a unique optimal strategy of HH that is given by the point of intersection of the

[16] If this condition is not fulfiled the minimal cost combination would be tangent to the area that fulfils the incentive compatibility constraints on a whole section represented by a part of the linear function (4) or (5) and not to a unique point.

linear combinations (4) and (5). Calculating this point of intersection leads to

$$\begin{aligned}
\theta^s_H(\theta^t_H-\theta^t_L)+\theta^s_L(\theta^s_H-\theta^s_L)-\frac{\theta^s_H}{\theta^t_L}t_{HH} &= \theta^s_L(\theta^s_H-\theta^s_L)+\frac{(\theta^t_H-\theta^t_L)\theta^s_L\theta^t_L}{\theta^t_H}-\frac{\theta^s_L}{\theta^t_H}t_{HH}\\
\Leftrightarrow \theta^s_H\theta^t_H(\theta^t_H-\theta^t_L)-\theta^s_L\theta^t_L(\theta^t_H-\theta^t_L) &= \frac{\theta^s_H\theta^t_H-\theta^s_L\theta^t_L}{\theta^t_L}t_{HH}\\
\Leftrightarrow (\theta^s_H\theta^t_H-\theta^s_L\theta^t_L)(\theta^t_H-\theta^t_L) &= \frac{\theta^s_H\theta^t_H-\theta^s_L\theta^t_L}{\theta^t_L}t_{HH}\\
\Leftrightarrow t^*_{HH} &= \theta^t_L(\theta^t_H-\theta^t_L).
\end{aligned}$$

Inserting this in equation (4) gives the first part of the equilibrium signal $s^* = \theta^s_L(\theta^s_H-\theta^s_L)$. □

Proof of Proposition 4:
The sequence of the proof of the partial separating PBE $(LL, HL, (LH, HH))$ is as follows:
First of all I find the optimal strategy for HL that prevents him from mimicing LL. Secondly I give the incentive compatibility constraint that prevents LL from mimicing (LH, HH) and vice versa. Thirdly, I give the incentive compatibility constraint that prevents HL from mimicing (LH, HH) and vice versa. Step two and three together result in an optimal strategy for (LH, HH).

In a PBE where LL is separated he has no icentive to signal. Thus, $(s^*_{LL}, t^*_{LL}) = (0,0)$. Then refering to the first step HL signals $(s^*_{HL}, t^*_{HL}) = (\theta^s_L(\theta^s_H-\theta^s_L), 0)$ to prevent LL from mimicing him. This strategy directly results from the separating PBE.

To make sure that in a second step LL does not mimic (LH, HH)

$$\begin{aligned}
w_{LL}-c_{LL}(s^*_{LL}, t^*_{LL}) &\geq w_{(LH,HH)}-c_{LL}(s_{(LH,HH)}, t^*_{(LH,HH)})\\
\Leftrightarrow \theta^s_L+\theta^t_L &\geq \frac{\alpha_{LH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_L+\frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_H+\theta^t_H\\
&\quad -\frac{s_{(LH,HH)}}{\theta^s_L}-\frac{t_{(LH,HH)}}{\theta^t_L}\\
\Leftrightarrow \frac{s_{(LH,HH)}}{\theta^s_L}+\frac{t_{(LH,HH)}}{\theta^t_L} &\geq \underbrace{\frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_H-(1-\frac{\alpha_{LH}}{\alpha_{LH}+\alpha_{HH}})\theta^s_L+\theta^t_H-\theta^t_L}_{\equiv C1_{(LH,HH)}}.
\end{aligned} \tag{6}$$

must hold.
Analogously, LH and therefore (LH, HH) does not mimic LL if

$$\begin{aligned}
w_{LL} - c_{LH}(s^*_{LL}, t^*_{LL}) &\leq w_{(LH,HH)} - c_{LH}(s_{(LH,HH),t_{LH,HH}}) \\
\Leftrightarrow \theta^s_L + \theta^t_L &\leq \frac{\alpha_{LH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_L + \frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_H + \theta^t_H \\
&\quad - \frac{s_{(LH,HH)}}{\theta^s_L} - \frac{t_{(LH,HH)}}{\theta^t_H} \\
\Leftrightarrow \frac{s_{(LH,HH)}}{\theta^s_L} + \frac{t_{(LH,HH)}}{\theta^t_H} &\leq \underbrace{\frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_H - (1 - \frac{\alpha_{LH}}{\alpha_{LH}+\alpha_{HH}})\theta^s_L + \theta^t_H - \theta^t_L}_{=C1_{(LH,HH)}}.
\end{aligned}$$

is fulfiled. Since signals can not be negative a necessary condition for the existence of the partial separating PBE is $C1_{(LH,HH)} > 0$. I will come to this later on.

To prevent HL from mimicing (LH, HH) (third step) the following condition must hold:

$$\begin{aligned}
w_{HL} - c_{HL}(s^*_{HL}, t^*_{HL}) &\geq w_{(LH,HH)} - c_{HL}(s_{(LH,HH)}, t_{(LH,HH)}) \\
\Leftrightarrow \theta^s_H + \theta^t_L - \frac{\theta^s_L(\theta^s_H - \theta^s_L)}{\theta^s_H} &\geq \frac{\alpha_{LH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_L + \frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_H + \theta^t_H \\
&\quad - \frac{s_{(LH,HH)}}{\theta^s_H} - \frac{t_{(LH,HH)}}{\theta^t_L} \\
\Leftrightarrow \frac{s_{(LH,HH)}}{\theta^s_H} + \frac{t_{(LH,HH)}}{\theta^t_L} \geq &
\end{aligned}$$

$$\underbrace{-(1 - \frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}})\theta^s_H + (1 + \frac{\alpha_{LH}}{\alpha_{LH}+\alpha_{HH}})\theta^s_L + \theta^t_H - \theta^t_L - \frac{(\theta^s_L)^2}{\theta^s_H}}_{\equiv C2_{(LH,HH)}} \qquad (7)$$

Analogously, to prevent HH and therefore (LH, HH) from mimicing HL

$$\begin{aligned}
w_{HL} - c_{HH}(s^*_{HL}, t^*_{HL}) &\leq w_{(LH,HH)} - c_{HH}(s_{(LH,HH)}, t_{(LH,HH)}) \\
\Leftrightarrow \theta^s_H + \theta^t_L - \frac{\theta^s_L(\theta^s_H - \theta^s_L)}{\theta^s_H} &\leq \frac{\alpha_{LH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_L + \frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}}\theta^s_H + \theta^t_H \\
&\quad - \frac{s_{(LH,HH)}}{\theta^s_H} - \frac{t_{(LH,HH)}}{\theta^t_H} \\
\Leftrightarrow \frac{s_{LH,HH}}{\theta^s_H} + \frac{t_{(LH,HH)}}{\theta^t_H} &\leq
\end{aligned}$$

$$\underbrace{-(1-\frac{\alpha_{HH}}{\alpha_{LH}+\alpha_{HH}})\theta^s_H+(1+\frac{\alpha_{LH}}{\alpha_{LH}+\alpha_{HH}})\theta^s_L+\theta^t_H-\theta^t_L-\frac{(\theta^s_L)^2}{\theta^s_H}}_{=C2_{(LH,HH)}}$$

must hold. A necessary condition for the existence of the partial separating PBE is again that $C2_{(LH,HH)} > 0$ is fulfiled. This condition is even stronger than $C1_{(LH,HH)}$ from above because

$$\begin{aligned} C2_{(LH,HH)} - C1_{(LH,HH)} &= -\theta^s_H + 2\theta^s_L - \frac{(\theta^s_L)^2}{\theta^s_H} \\ &= \frac{-(\theta^s_H)^2 + 2\theta^s_H\theta^s_L - (\theta^s_L)^2}{\theta^s_H} \\ &= -\frac{(\theta^s_H - \theta^s_L)^2}{\theta^s_H} < 0 \end{aligned}$$

holds. Although $C2_{(LH,HH)} < C1_{(LH,HH)}$ is fulfiled one can not directly see if equation (6) or equation (7) is the stronger condition because of the different LHS. If you compare both conditions you find that the relationship depends on the exact parameter values. However, I show that the optimal - cost minimal - behavior for type LH and HH is the same regardless whether equation (6) or equation (7) is the stronger condition.

Thus assume that equation (6) is stronger than equation (7) then $t_{(LH,HH)} = \theta^t_L C1_{(LH,HH)} - \frac{\theta^t_L}{\theta^s_L} s_{(LH,HH)}$ holds. This in mind costs of LH are

$$\frac{s_{(LH,HH)}}{\theta^s_L} + \frac{t_{(LH,HH)}}{\theta^t_H} = \frac{1}{\theta^s_L}\underbrace{(1-\frac{\theta^t_L}{\theta^t_H})}_{>0} s_{(LH,HH)} + \frac{\theta^t_L}{\theta^t_H} C1_{(LH,HH)}.$$

Since costs increase in $s_{(LH,HH)}$ the optimal strategy of LH is $s_{(LH,HH)} = 0$. Analogously, costs of HH are

$$\frac{s_{(LH,HH)}}{\theta^s_H} + \frac{t_{(LH,HH)}}{\theta^t_H} = (\frac{1}{\theta^s_H} - \frac{\theta^t_L}{\theta^s_L\theta^t_H}) + \frac{\theta^t_L}{\theta^t_H} C1_{(LH,HH)}.$$

If $\frac{1}{\theta^s_H} - \frac{\theta^t_L}{\theta^s_L\theta^t_H} \leq 0$ holds the optimal strategy is to maximise $s_{(LH,HH)}$. However, then the partial separating PBE is destroyed. Type LH and HH do not play the same strategy. Therefore $\frac{1}{\theta^s_H} - \frac{\theta^t_L}{\theta^s_L\theta^t_H} \geq 0$ must hold to ensure the described PBE. If equation (6) is the stronger condition one $\theta^s_L\theta^t_H \geq \theta^s_H\theta^t_L$ becomes a necessary condition of the partial separating PBE.

Now assume that instead of equation (6) equation (7) is the stronger condition then $t_{(LH,HH)} = \theta^t_L C2_{(LH,HH)} - \frac{\theta^t_L}{\theta^s_H} s_{(LH,HH)}$ holds and costs of type LH are

$$\begin{aligned}\frac{s_{(LH,HH)}}{\theta^s_L} + \frac{t_{(LH,HH)}}{\theta^t_H} &= (\frac{1}{\theta^s_L} - \frac{\theta^t_L}{\theta^s_H \theta^t_H}) s_{(LH,HH)} + \frac{\theta^t_L}{\theta^t_H} C2_{(LH,HH)} \\ &= \underbrace{(\frac{\theta^s_H \theta^t_H - \theta^s_L \theta^t_L}{\theta^s_H \theta^t_L \theta^t_H})}_{>0} s_{(LH,HH)} + \frac{\theta^t_L}{\theta^t_H} C2_{(LH,HH)}.\end{aligned}$$

Again it is optimal for type LH to play $s_{(LH,HH)} = 0$. Analogously, costs of type HH are

$$\frac{s_{(LH,HH)}}{\theta^s_H} + \frac{t_{(LH,HH)}}{\theta^t_H} = \frac{1}{\theta^s_H}(1 - \frac{\theta^t_L}{\theta^t_H}) s_{(LH,HH)} + \frac{\theta^t_L}{\theta^t_H} C2_{(LH,HH)}.$$

As costs increase in $s_{(LH,HH)}$ type HH sets $s_{(LH,HH)} = 0$.

Summarising, under both assumption $s^*_{(LH,HH)} = 0$ is an optimal strategy for both pooling types. This reduces condition (6) to $t_{(LH,HH)} = \theta^t_L C1_{LH,HH}$ and condition (7) to $t_{(LH,HH)} = \theta^t_L C2_{(LH,HH)}$. With $C1_{(LH,HH)} > C2_{(LH,HH)}$ from the above condition (6) becomes the crucial condition for the existence of the partial separating PBE. The equilibrium strategy of (LH, HH) is $(s^*_{(LH,HH)}, t^*_{(LH,HH)}) = (0, \theta^t_L C1_{(LH,HH)})$. A necessary condition for the existence of the equilibrium is $\theta^s_L \theta^t_H > \theta^s_H \theta^t_L$.

Finally, the proof of the second partial PBE, i.e. of $(LL, LH, (HL, HH))$ is analogous and is therefore not specified here. □

References

Allgood, Sam and Wiliam B. Walstad (2005). Views of teaching and research in economics and other disciplines. *American Economic Review Papers and Proceedings 95*(2), 177–183.

Armstrong, Mark and Jean-Charles Rochet (1999). Multi-dimensional screening: A user's guide. *European Economic Review 43*, 959–979.

Becker, William E. (1975). The university professor as a utility maximizer and producer of learning, research, and income. *Journal of Human Ressources 10*, 109–115.

Becker, William E. (1979). Professorial behavior given a stochastic reward structure. *The American Economic Review 69*(5), 1010–1017.

Becker, William E. and Peter E. Kennedy (2005). Does teaching enhance research in economics? *American Economic Review Papers and Proceedings 95*(2), 172–176.

Cho, In-Koo and David M. Kreps (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics 102*(2), 179–221.

De Fraja, Gianni and Paola Valbonesi (2008). The design of the university system. CEPR Discussion Paper no. 7038.

Del Rey, Elena (2001). Teaching versus research: A model of state university competition. *Journal of Urban Economics 49*, 356–373.

Engers, Maxim (1987). Signalling with many signals. *Econometrica 55*(3), 663–674.

Gautier, Axel and Xavier Wauthy (2007). Teaching versus research: The role of internal financing rules in multi-department univeristies. *European Economic Review 51*, 273–295.

Gottlieb, Esther E. and Bruce Keith (1997). The academic research-teaching nexus in eight advanced-industrialized countries. *Higher Education 34*, 397–420.

Hattie, John and H. W. Marsh (1996). The relationship between research and teaching: A meta-analysis. *Review of Educational Research 66*(4), 507–542.

Kim, Jeong-Yoo (2007). Multidimensional signaling in the labor market. *Manchester School 75*(1), 64–87.

Mitchell, John E. and Douglas S. Rebne (1995). Nonlinear effects of teaching and consulting on academic research productivity. *Socio-Economic Planning Sciences 29*(1), 47–57.

Rochet, Jean-Charles and Martine Qunizii (1985). Multidimensional signalling. *Journal of Mathematical Economics 14*, 261–284.

Spence, Michael (1973). Job markt signalling. *The Quarterly Journal of Economics 87*(3), 355–374.

Spence, Michael (1974). Competitive and optimal responses to signals: An analysis of efficiency and distribution. *Jounral of Economic Theory 7*, 296–332.

Walckiers, Alexis (2008). Multi-dimensional contracts with task-specific productivity: An application to universities. *International Tax and Public Finance 15*, 165—198.