

# COMPARING PUBLIC-PRIVATE SCHOOL MANAGEMENT THROUGH A NEW EDUCATIONAL MALMQUIST INDEX APPROACH

Eva Crespo Cebada  
(University of Extremadura)  
Francisco Pedraja Chaparro  
(University of Extremadura)  
Daniel Santín González  
(Complutense University of Madrid)

November 2009

## **Abstract**

It is well known in the educational literature that public and private-voucher schools show different production technologies and present differences in the productivity of their educational inputs due to management issues. On the one hand the students attending public schools differ in their socioeconomic characteristics from students in the private-voucher (private management and funded by the government) ones. In this paper we propose a new non-parametric Educational Malmquist index approach in order to analyse total factor productivity changes and divergences between publicly financed schools when only a pseudo-panel database is available. To do this we use the Basque Country data from PISA 2003 and 2006. The results suggest a higher productivity change for public schools, due to technical efficiency improvements, whereas the technology progress is higher in private-voucher schools within 2003-2006.

*Keywords:* Efficiency, Productivity, Education, PISA, Malmquist.

## 1. INTRODUCTION

One of the main goals in the field of economics of education is to analyze the inefficiency behaviours in the learning process. The sources of inefficiency may be due to multiple reasons such as the way in which resources are organized and managed, the motivation of the agents involved in the process or the structure itself of the educational system (Nechyva, 2000; Woessman, 2001).

The recently increase of national and international programs to evaluate the scholar achievement during last decades shows the higher policy concern about educational performance. Hence, last years some international projects have been developed in order to evaluate the educational achievements in which are considered the vehicular disciplines: Science, Mathematics and Lecture. The most important international programs are TIMSS (*Third International Mathematics and Science Study*), PISA (*Programme for International Student Assessment*) and PIRLS (*Progress in International Reading Literacy Study*) although many countries perform their own evaluations e.g. the *National Assessment of Educational Progress* (NAEP) in the United States.

The main advantage of these programs is that provide an external evaluation of educational results with the aim of identifying causes of school failure allowing to policy makers and school principals to go into their management strengths and weakness in depth. However, the comparison of students or schools behaviours along the time using these international studies is not possible due to participant schools and students differ from one wave to another.

In order to tackle the inefficiency measurement issue in education many studies have used non parametric Data Envelopment Analysis (Bessent and Bessent, 1980; Charnes, Cooper, and Rhodes, 1981 and Bessent et al., 1982<sup>1</sup>) and other parametric methodologies (Christensen, Jorgenson, and Lau, 1971; Gyimah-Brempong and Gyapong, 1992; Deller and Rudnicki, 1993, Grosskopf et al., 1997, Perelman and Santín, 2008).

In this paper we propose the use of the well known Malmquist Index in order to obtain a measurement of productivity divergences between public and private-voucher schools in two time periods (2003-2006). With this aim we provide an empirical application to Basque Country educational data from the *Programme for International Student Assessment* (PISA),

---

<sup>1</sup> For an empirical survey of frontier efficiency techniques in education, see Worthington (2001).

implemented in 2003 and 2006 by the Organization for Economic Cooperation and Development (OECD). PISA includes a wide variety of background information on the students collected by student questionnaires and about schools resources, (for an extensive review see OECD, 2007a, 2007b and 2009).

The paper is organized as follows. Section 2 provides an overview about the Malmquist Index together with our estimation strategy. In Section 3 data set and selected inputs and outputs are described. Section 4 provides results and a discussion of our empirical analysis and the final section offers some conclusions and future lines for research.

## 2. METHODOLOGY

Malmquist Index was proposed by Caves, Christensen and Diewet (1982) with the aim of measuring the productivity changes within two time periods as the distance between a decision making unit (DMU) and the frontier for each period. The index is built using different Data Envelopment Analysis (DEA) programs<sup>2</sup>, so no assumptions, beyond monotonicity and convexity, about the production technology are required. Hence, it is especially attractive in the educational context, where multiple inputs and output are involved and prices are unknown or difficult to estimate.

The Malmquist Index provides a measure of the total productivity factors (TPF) evolution and their components along the time, so the TPF is explained by the efficiency change, which is known as catch-up effect, and the technology change (frontier shift). Figure 1 illustrates the TFP change for a DMU in two periods.

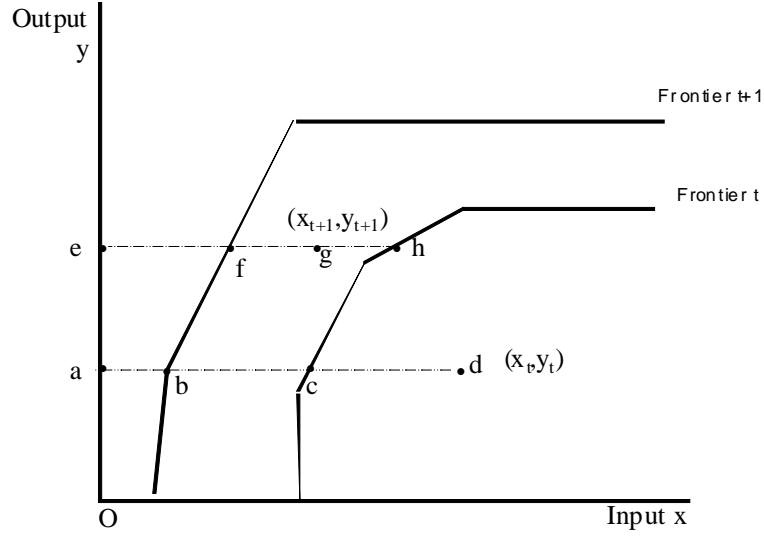
In Figure 1 DMU d (g) employs in period t (t+1)  $x_t$  ( $x_{t+1}$ ) units of input to produce  $y_t$  ( $y_{t+1}$ ) units of outputs. Through these quantities the Malmquist index measures the catch-up effect as the ratio  $\frac{ef}{eg} / \frac{ac}{ad}$ . When the index is greater (less) than one the DMU improves

(make worse) its efficiency. The frontier or technology shift is denoted by  $\sqrt{\frac{ac}{ab} \cdot \frac{bh}{bf}}$  where values greater (less) than one implies a production frontier upward (downward) movement

---

<sup>2</sup> The analytic framework is described in detail in the Appendix.

**Figure 1: Productivity change with one input and one output**



To formalize the index we first assume constant returns to scale (CRS). In defining a vector of inputs  $x = (x_1, \dots, x_K) \in \mathfrak{R}^{K^+}$  and a vector of outputs  $y = (y_1, \dots, y_M) \in \mathfrak{R}^{M^+}$ , a feasible multi-input multi-output production technology for a period of time  $t$  ( $t = 1, \dots, T$ ) can be defined using the output possibility set  $P^t(x^t)$ , which can be produced using the input vector  $x^t$ :  $P^t(x^t) = \{y^t: x^t \text{ can produce } y^t\}$ , which is assumed to satisfy the set of axioms described in Färe and Primont (1995). This technology can also be defined as the output distance function proposed by Shephard (1970):

$$D^t(x^t, y^t) = \inf \{ \theta : \theta > 0, (x^t, y^t / \theta) \in P^t(x^t) \}$$

If  $D^t(x^t, y^t) \leq 1$ , then  $(x^t, y^t)$  belongs to the production set  $P^t(x^t)$ . In addition,  $D^t(x^t, y^t) = 1$ , if  $y^t$  is located on the outer boundary of the output possibility set.

In order to define the Malmquist productivity index proposed by Färe et al. (1994), we assume the distance function,  $D^t$ , and the inputs and outputs endowments  $x^t, y^t$  for every period of time,  $t$ , ( $t = 1, \dots, T$ ). The index can be represented as follows:

$$M(x^{t+1}, y^{t+1}, x^t, y^t) = \frac{D^{t+1}(x^{t+1}, y^{t+1})}{D^t(x^t, y^t)} * \left[ \left( \frac{D^t(x^{t+1}, y^{t+1})}{D^{t+1}(x^{t+1}, y^{t+1})} \right) \left( \frac{D^t(x^t, y^t)}{D^{t+1}(x^t, y^t)} \right) \right]^{1/2} = TEC * TC$$

where a higher than one index implies productivity improvements and lower than one productivity losses<sup>3</sup>.

Such as we may see in the expression above, the index can be decomposed into two items. The first one, the technical efficiency change (TEC) shows improvements on efficiency in period t+1 if  $TEC > 1$  and the opposite for  $TEC < 1$ , being  $TEC = 1$  no changes on technical efficiency. The last one represents the technological change (TC), which may have different direction from TEC.

Furthermore the index may be calculated assuming constant returns to scale (CRS) and variable returns to scale (VRS) which allow us to decompose the efficiency change into pure efficiency and scale efficiency changes as shows the following expression:

$$TEC = \frac{D_o^{t+1}(X^{t+1}, Y^{t+1})}{D_o^t(X^t, Y^t)} = \frac{D_{VRS}^{t+1}(X^{t+1}, Y^{t+1})}{D_{VRS}^t(X^t, Y^t)} \frac{D_{CRS}^{t+1}(X^{t+1}, Y^{t+1})}{D_{CRS}^t(X^t, Y^t)} = PEC * SEC$$

being the first item, the pure efficiency change (PEC) with respect to variable returns to scale frontier and the second one, the scale efficiency change (SEC) that reflects changes between both CRS and VRS frontiers.

Therefore, following Ray and Desli (1997), the Malmquist Index considering VRS, comprises three elements: the pure efficiency change (PEC), the scale efficiency change (SEC) and the technology change (TC).

$$M(x^{t+1}, y^{t+1}, x^t, y^t) = \frac{D^{t+1}(x^{t+1}, y^{t+1})}{D^t(x^t, y^t)} * \left[ \left( \frac{D^t(x^{t+1}, y^{t+1})}{D^{t+1}(x^{t+1}, y^{t+1})} \right) \left( \frac{D^t(x^t, y^t)}{D^{t+1}(x^t, y^t)} \right) \right]^{1/2} = TEC * TC$$

The purpose of our study is to analyse productivity differences among public and private-voucher schools using the Malmquist Index approach. Thus, we propose a non-parametric Educational Malmquist in order to obtain productivity divergences between public

---

<sup>3</sup> This productivity index is the geometric mean of two productivity index, where the first one takes t period as reference and the second one t+1, avoiding arbitrary selection in the period of reference.

and private-voucher schools for the same year instead of making the comparison across the time<sup>4</sup>.

The expression for this Educational Malmquist, where “C” and “P” super-index indicate private-voucher and public schools respectively, is as follows:

$$M_i = \frac{D_{\text{VRS}}^C(X^C, Y^C)}{D_{\text{VRS}}^P(X^P, Y^P)} \frac{D_{\text{CRS}}^C(X^C, Y^C)}{D_{\text{CRS}}^P(X^P, Y^P)} \cdot \left[ \frac{D^C(x^P, y^P)}{D^P(x^P, y^P)} \times \frac{D^C(x^C, y^C)}{D^P(x^C, y^C)} \right]^{\frac{1}{2}}$$

where index  $i$  indicates the time period.

The expression above shows productivity differences in one year between public and private-voucher schools. We may distinguish three components, the first one refers to pure efficiency differences among them, which are the differences in distances for each management system to its own VRS frontier, the second item is the scale efficiency divergence, that indicates how separated are both CRS and VRS frontiers for public and private-vouchers schools respectively, and the last one is the technology difference between public and private-voucher schools.

This approach allows us to compare mean productivities between educational systems in order to analyse efficiency and technology differences from both public and private-voucher schools. So, if the index is higher than one implies that private-voucher schools are more productive than public ones.

Figure 2 illustrates these concepts in a simple one output-one input setting in one period. Let assume that the frontier  $S_{\text{VRS}}^i$  represents variable returns to scale (VRS) technology and the constant returns to scale (CRS) technology is indicated by the line  $S_{\text{CRS}}^i$ , where the superindex indicates the school ownership, public (P) and private-voucher (C). Moreover the average inefficient public (private-voucher) school consumes  $x^P$  ( $x^C$ ) input and produces  $y^P$ , ( $y^C$ ) output.

---

<sup>4</sup> To perform this analysis we need a pseudo-panel database as PISA or TIMSS, where sampled schools are representative from public and private-voucher educational systems.

Then the Educational Malmquist Index expressed as the product of these three components would be:

$$M = \frac{(Oe/Oq)}{(Od/Op)} \cdot \left[ \frac{\frac{(Oc/Oq)}{(Oe/Oq)}}{\frac{(Ob/Op)}{(Of/Op)}} \right] \cdot \sqrt{\left[ \frac{(Og/Oq)}{(Oc/Oq)} \cdot \frac{(Ob/Op)}{(Oa/Op)} \right]}$$

As we mention above, divergences in productivity may be explained by differences between public and private-voucher schools in three components: the pure efficiency difference (PED), which indicates the distance from a private-voucher school to its VRS frontier respecting to the distance from a public school to the same frontier, is given by the ratio:

$$PED = \frac{(Oe/Oq)}{(Od/Op)}$$

The scale efficiency difference (SED), which shows how separated are both CRS and VRS for a private-voucher school with respect to the same distance for a public school, is given by:

$$SED = \frac{\frac{(Oc/Oq)}{(Oe/Oq)}}{\frac{(Ob/Op)}{(Of/Op)}}$$

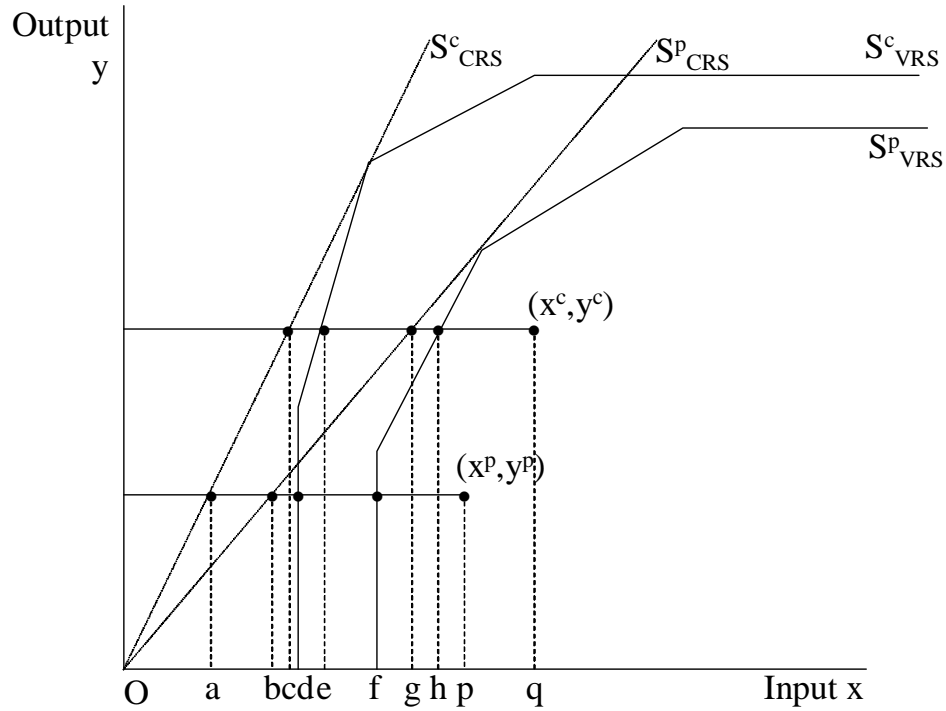
Finally, technological divergences (TD) among schools, referring to CRS frontiers, are showed by:

$$TD = \sqrt{\left[ \frac{(Og/Oq)}{(Oc/Oq)} \cdot \frac{(Ob/Op)}{(Oa/Op)} \right]}^5$$

---

<sup>5</sup> In this simple graphical example the two components of TC are identical, but this will not in general be the case.

**Figure 2: Productivity divergences between public and private-voucher schools**



Furthermore, we are interesting in analysing productivity changes within two periods using the ratio of two Educational Malmquist expressions. Then the productivity deviation of private-voucher schools respect to public one within  $t$  and  $t+1$  is as follows:

$$MC = \frac{M_t}{M_{t+1}}$$

This expression indicates productivity gains for public schools when  $MC > 1$  or productivity losses when  $MC < 1$ . Similarly, the pure efficiency change (PEC), the scale efficiency change (SC) and the technology changes (TC) between public and private-voucher schools in the two periods may be explained by the following ratios:

$$PEC = \frac{ED_t}{ED_{t+1}} \quad TC = \frac{TD_t}{TD_{t+1}} \quad SC = \frac{SED_t}{SED_{t+1}}$$

where a higher than one ratio shows public schools are more efficiency and advanced technologically than private-voucher schools are.



### 3. DATASET AND VARIABLES

For the illustration purpose of our study we only use Basque Country school data from PISA 2003 and 2006 database. There are two main reasons to choose the Basque Country as the object of our analysis. Firstly, although ten regions took part in PISA 2006 only three of them were in PISA 2003 (Castile-Leon, Catalonia and the Basque Country). Secondly the Basque Country is the region with a highest proportion of private-voucher schools, which guarantees a similar number of public and private voucher schools. Specifically, we have data about 73 (83) private-voucher schools and 54 (64) public schools in PISA 2003 (2006)<sup>6</sup>.

One of the main advantages of the PISA study is that it does not evaluate cognitive abilities or skills through using one single score but each student receives a score in each test within a continuous scale. In this way, PISA attempts to collect the effect of particular external conditioning factors not depending on the students when taking the test, namely being ill or becoming very nervous, among other random factors. Furthermore, it also involves that measurement error in education is not independent from the position of the student in the distribution of results. Precisely, students with very low or high results have higher associated measurement errors and higher asymmetry in error distribution.

Likewise, PISA also collects an extensive dataset on these variables through two questionnaires: one completed by the students themselves and another one filled out by principals. From these data, it is possible to extract a great amount of information referred to the main determining factors of educational performance represented by variables associated to familiar and educational environments as well as to school management and educational supply.

#### *Outputs and plausible values*

The true output as result of an individual education is very difficult to measure empirically due to its inherent intangibility. Education does not only consist of the ability of repeating information and answering questions, but it also involves the skills to interpret the information and learn how to behave in the society. Unfortunately, it is really hard to measure all of them. But perhaps, according to Hoxby (2000), the most important reason could be that both policy makers and parents use this criterion to evaluate the educational output and its subsequent information to choose the school for their children and even their place of residence.

---

<sup>6</sup> Finally after controlling for missing data we perform the analysis choosing randomly 51 public and private-voucher schools in years 2003 and 2006. After a number of proofs (resampling) we concluded that our results are highly robust using plausible values as output. In databases where plausible values are not available the use of the bootstrap will be a possible solution to different sample sizes for public and private-voucher schools.

In this study we use the results obtained by students in the three competences evaluated in PISA (math, reading comprehension and sciences) as school output. As it has already been mentioned, the study uses the concept of plausible values to measure the performance of students, since measures in these subjects have a wide margin of error due to the fact that the measuring concept is abstract and the measure is subject to the special circumstances of students and their environment on the date of their exams. These values are random values obtained from the distribution function of results estimated from the answers in each test. They can be interpreted as a representation of the ability range for each student<sup>7</sup> (Wu and Adams, 2002).

Plausible values in the three tests are used as outputs in the efficiency analysis. In order to obtain correct results and avoid problems of bias in estimations it will be necessary to calculate five different Malmquist index for each trio of plausible values and take the mean value afterwards, instead of using mean values to obtain one Malmquist index (OCDE, 2005).

**Table 1: Descriptive statistics of outputs and inputs**

	2003		2006	
	Public	Private-voucher	Public	Private-voucher
OUTPUTS				
Math_PV1	483.34	509.98	479.85	517.07
Math_PV2	482.30	509.83	471.45	511.66
Math_PV3	481.66	509.65	480.74	507.78
Math_PV4	483.75	510.73	478.69	511.28
Math_PV5	511.28	511.28	511.28	511.28
Read_PV1	472.71	511.98	471.14	497.43
Read_PV2	470.36	508.83	474.90	497.45
Read_PV3	472.48	508.43	474.50	490.09
Read_PV4	474.29	510.20	471.71	496.51
Read_PV5	496.51	496.51	496.51	496.51
Science_PV1	461.20	499.04	475.41	499.12
Science_PV2	462.68	495.61	475.22	498.65
Science_PV3	462.34	496.57	478.84	494.71
Science_PV4	463.65	497.37	476.27	495.74
Science_PV5	495.74	495.74	495.74	495.74

<sup>7</sup> For a review of plausible values literature see Mislevy *et al.* (1992). For a concrete Studio of Rasch model and how obtain feasible values in PISA, see OECD (2005).

INPUTS				
PARED	11.97	11.30	12.16	12.19
HISEI	45.65	43.64	42.08	47.69

### *Inputs*

In order to calculate the Malmquist index we have used two different inputs that are directly involved with learning (HISEI and PARED). HISEI is the index of highest parental occupation status according to International Socio-economic index of Occupational Status (ISEI, Ganzeboom *et al.*, 1992) and PARED is the index of highest level of parental education in number of years of education according to the *International Standard Classification of Education* (ISCED, OECD, 1999). According to Hanushek (2003) it is difficult to find a systematic relationship between more school resources devoted to schools and achievement and for this reason we only include mean family characteristics of students as inputs.

## 4. RESULTS

This section presents the main results obtained in our analysis. Our methodology allows us comparing public and private-voucher schools productivity changes between 2003 and 2006. Table 2 reports the results after applying this methodology, where public schools are pseudo-considered as period t and private-voucher as t+1.

**Table 2: Educational Malmquist Index**

Technical Efficiency Divergence	Technology Divergence	Pure Efficiency Divergence	Scale Efficiency Divergence	Educational Malmquist Index
2003				
1.173 (0.017)	0.955 (0.016)	1.013 (0.011)	1.158 (0.022)	1.120 (0.003)
2006				
0.765 (0.024)	1.279 (0.041)	1.064 (0.037)	0.719 (0.019)	0.977 (0.003)

The results show that in 2003 private-voucher schools were 12% more productive than the public schools. However the private-voucher schools productivity falls down in 2006, being a 2.3% less productive than public schools. So, public schools present around 14% productivity improvement within 2003-2006 periods, which may be partly explained by the wide improve on public schools efficiency.

Furthermore, as we appreciate in Table 2, public schools undergo an efficiency increase respecting to private-voucher ones. In fact, whereas public schools were 17% less efficiency in 2003, in the second period they experiment an important improvement, being 23.5% more efficient than private-voucher schools are. Nevertheless, the apparently reason for this efficiency growth is the distance reduction between both CRS and VRS frontiers for public schools, so the scale efficiency for public schools is around 28% higher, since private-voucher schools present a higher pure efficiency in both periods. However, technology differences present the opposite sign, so technological progress for private schools in 2006 is about 27.9% better than public ones.

Until now we have analysed Basque Country schools' divergences on productivity and their components in two time periods. Nevertheless, with the aim of going into productivity changes within this period in depth, we calculate the ratios explained in section 2 and the results are showed in Table 3.

**Table 3: Productivity gains between public and private-voucher schools within 2003-2006**

Technical Efficiency Change	Technology Change	Pure Efficiency Change	Scale Efficiency Change	Educational Malmquist Index
1.535 (0.045)	0.747 (0.023)	0.953 (0.032)	1.612 (0.058)	1.146 (0.003)

These results enforce the Educational Malmquist Index results commented above, and bears strong implications for the educational policies, even more taking into account that educational competences in Spain are decentralize, so educational decisions are taken at regional level.

The first relevant result we appreciate from table 3 is that public schools progress in productivity around 14.6% more than private-voucher ones within 2003-2006<sup>8</sup>. Similar results were obtained by Kirjavainen and Loikkanen (1998) for Finland, Newhouse and Beegle (2006) for Indonesia and Calero and Escardibul (2007) for Spain using different methods.

Moreover, as expected, the technical efficiency is around 53.5% higher for public schools, although their technology progress is about 25.3% smaller respecting to private-voucher schools. All these conclusions are relevant bearing in mind the students' characteristics in public schools, where there is a higher proportion of immigrant students, higher number of repeated or lower socio-economic background. All these factors are relevant to explain the student inefficiency as shows some studies Chiswick and Debburman (2004), Calero and Waigrais (2009) for the immigrant effect and Coleman et al. (1966) and Hanushek (1997, 2003) referring to the student background.

From this result we can conclude that better inputs (average family background) in private-voucher schools deteriorate the educational productivity. It seems that private-voucher schools are more technological advanced and more purely efficient than their public counterparts but they are subject to strong decreasing returns to scale that is deteriorating the educational productivity.

## 5. CONCLUSIONS

Malmquist Index methodology is widely used in the literature with the aim of measuring the productivity growth within two time periods as the distance between each DMU and the frontier for each period. This methodology allows to decomposing the productivity change into three components: the technical efficiency, the scale efficiency and the technology change. To perform a Malmquist index we need a panel database.

In this paper, we propose a new Educational Malmquist in order to compare productivity divergences between public and private-voucher schools across the time when only a pseudo-panel database is available.

The results show that there are relevant differences in schools productivity. Then, although private-voucher schools are 12% more productive in 2003, their productivity decrease 2006, being 2.3% less productive than public schools.

---

<sup>8</sup> With the aim of obtaining the productivity gains within the period 2003-2006 we calculate the educational Malmquist Index ratio.

Moreover, in order to analyse productivity changes among both ownership schools along that period some ratios were calculated. The results show that public schools productivity progress around 14.6% more than private-voucher ones within 2003-2006, which may be partly explained by the wide improvement on public schools efficiency (53.5%), since their technology progress is about 25.3% lower than private-voucher schools.

These conclusions should be interpreted cautiously, since they are referred to a particular context and time however their implications are very relevant for the design of educational-policy in the Basque Country. Our results point out that more similar average family characteristics among schools will increase the productivity in the educational Basque Country system.

## REFERENCES

Bessent, A.M. and Bessent, E.W. (1980): "Determining the Comparative Efficiency of Schools Through Data Envelopment Analysis", *Educational Administration Quarterly*, 16 (2), pp. 57-75.

Bessent, A.; Bessent, W.; Kennington, J. and Reagan, B. (1982): "An application of mathematical programming to assess productivity in the Houston independent school district", *Management Science*, 28, 1355-1367.

Calero, J. and Escardibul, J. O. (2007): "Evaluación de servicios educativos: el rendimiento en los centros públicos y privados medido en PISA-2003", *Hacienda Pública Española*, nº 183 (4/2007), pp. 33-66.

Calero, J. and Waisgrais, S. (2009): "Rendimientos educativos de los alumnos inmigrantes: identificación de la incidencia de la condición de inmigrante y de los peer-effects", *paper presented in XVI Encuentro de Economía Pública*, Granada, february, 2009.

Caves, D.W.; Christensen, L. R. and Diewert, W. E. (1982). "The economics theory of index numbers and the measurement of input, output and productivity", *Econometrica*, 50, 1393-1414.

Charnes, A.; Cooper, W. W. and Rhodes, E. (1981): "Evaluating program and managerial efficiency: an application of data envelopment analysis to Program Follow Through", *Management Science*, 27, 668-697.

Chiswick, B. and Debburman, N. (2004): "Educational attainment: analysis by immigrant generation", *Economics of Education Review*, 23(4), 361-379.

Christensen, L.R.; Jorgenson, D. W. and Lau, L. J. (1971): "Conjugate Duality and the Transcendental Logarithmic Production Function", *Econometrica*, 39, pp. 255-256.

Coleman, J.; Campbell, E. Q.; Hobson, C. F.; McPartland, J. and Mood, A. M. (1966): *Equality of Educational Opportunity*, Washington, U.S, Office of Education.

Deller, S.C. and Rudnicki, E. (1993): "Production Efficiency in Elementary Education. The Case of Maine Public School", *Economics of Education Review*, vol. 12, nº 1, págs. 45-57.

Färe, R.; Grosskopf, S. and Lovell, C.A.K. (1994): *Production Frontiers*. Cambridge University Press.

Ganzeboom, H.; De Graaf, P.; Treiman, J. and De Leeuw, J. (1992): “A standard internacional socio-economic index of ocupational status”, *Social Science Research*, 21 (1), pp. 1-56.

Gyimah-Brempong, K. and Gyapong, A. (1992): Elasticities of factor substitution in the production of education, *Economics of Education Review*, 11. 205–217.

Grosskopf, S.; Hayes, K.; Taylor, L. and Weber, W. (1997): “Budget-constrained frontier measures of fiscal equality and efficiency in schooling”, *Review of Economics and Statistics*, 79 (1), 116-124.

Hanushek, E. A. (1997): “Assessing the effects of school resources on student performance: An update”, *Educational Evaluation and Policy Analysis*, 19, 141-164.

Hanushek, E. A. (2003): “The failure of input based schooling policies,” *The Economic Journal*, 113, 64-98.

Hoxby, C. M. (2000): “The effects of class size on student achievement: new evidence from population variation”. *Quarterly Journal of Economics*, 115, 1239-1285.

Mislevy, R. J.; Beaton, A. E.; Kaplan, B. and Sheehan, K. M. (1992): “Estimating population characteristics form sparse matrix samples of item responses”, *Journal of Educational Measurement* 29, pp.133-161.

Nechyba, T. J. (2000): “Mobility targeting and private-school vouchers”, *American Economic Review*, 90 (1), 130-146.

Newhouse, D. and Beegle, K. (2006): “The Effect of School Type on Academic Achievement: Evidence from Indonesia”, *Journal of Human Resources*, 41(3): 529–557.

OECD (2005): *PISA 2003 Data Analysis Manual, SPSS users*, Organisation for Economic Co-operation and Development, Paris.



OECD (2007): *PISA 2006: Science competencies for tomorrow's world*, Paris.

OECD (2009): *PISA 2006 Technical Report*, OECD. Paris.

Ray, S. and Desli, E. (1997). Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries: Comment. *American Economic Review*, 87 (5), 1033-1039.

Woessman, L. (2001): "Why students in some countries do better". *Education Matters*, vol. 1(2), pp. 67-74.

Worthington, A. C. (2001): "An Empirical Survey of Frontier Efficiency Measurement techniques in Education", *Education Economics*, vol. 9, n° 3.

Wu, M. and Adams, R. J. (2002): "Plausible Values – Why They Are Important", International Objective Measurement Workshop, New Orleans.

## APPENDIX

The application of DEA to the Malmquist index requires the solution of four linear programming problems for each of the  $n$  units under investigation that corresponds to the four required distance functions in order to take in mind both periods and both CRE and VRS technologies.

The linear programming, assuming constant returns to scale for public schools and using data from private-voucher schools, can be considered solving for  $n$  units the problem:

$$\begin{aligned} & [D_i^P(X_C, Y_C)]^{-1} = \max_{\phi, \lambda} \phi \\ \text{subject to} \quad & -\phi y_{i(C)} + Y_P \lambda \geq 0 \\ & x_{i(C)} - X_P \lambda \geq 0 \\ & \lambda \geq 0 \end{aligned}$$

where  $x_{i,C}$  and  $y_{i,C}$  are the vectors of inputs and outputs associated with the school  $i$  and  $\lambda$  is a flexible vector of weights to be applied to the matrices  $X_C$  and  $Y_C$ . The parameter  $n$  indicates the maximum proportion by which all outputs of school  $i$  can be expanded such that  $(x_{i,C}; y_{i,C} / n)$  remains feasible.

In a similar way, the linear programming assuming constant returns to scale for private-voucher schools and using data from public ones can be considered solving for  $n$  units the problem:

$$\begin{aligned} & [D_i^C(X_P, Y_P)]^{-1} = \max_{\phi, \lambda} \phi \\ \text{subject to} \quad & -\phi y_{i,P} + Y_C \lambda \geq 0 \\ & x_{i,P} - X_C \lambda \geq 0 \\ & \lambda \geq 0 \end{aligned}$$

Both remaining DEA specifications considering variable returns to scale, considering data and technology from public and private-voucher schools respectively, are:

$$[D_i^P(X_P, Y_P)]^{-1} = \max_{\phi, \lambda} \phi$$

subject to

$$\begin{aligned} -\phi y_{i,P} + Y_P \lambda &\geq 0 \\ x_{i,P} - X_P \lambda &\geq 0 \\ \lambda &\geq 0 \end{aligned}$$

$$[D_i^C(X_C, Y_C)]^{-1} = \max_{\phi, \lambda} \phi$$

subject to

$$\begin{aligned} -\phi y_{i,C} + Y_C \lambda &\geq 0 \\ x_{i,C} - X_C \lambda &\geq 0 \\ \lambda &\geq 0 \end{aligned}$$